國立高雄第一科技大學
National Kaohsiung First University of Science and Technology

Gaussian Mixture Models (GMM) for Background Subtraction

Chris Stauffer and W. E. L. Grimson

IEEE. Intl. Conf. on Computer Vision and Pattern Recognition, 1999

Speaker: Shih-Shinh Huang

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C. Stauffer and W. Grimson, "Adaptive background mixture models for real time tracking", *Proc. of IEEE Int. Conf. on CVPR, Vol.2,*pp.246-252, June 1999.

Outline

- Introduction
- Gaussian Mixture Model
- Modeling Process
- Subtraction Process

Introduction

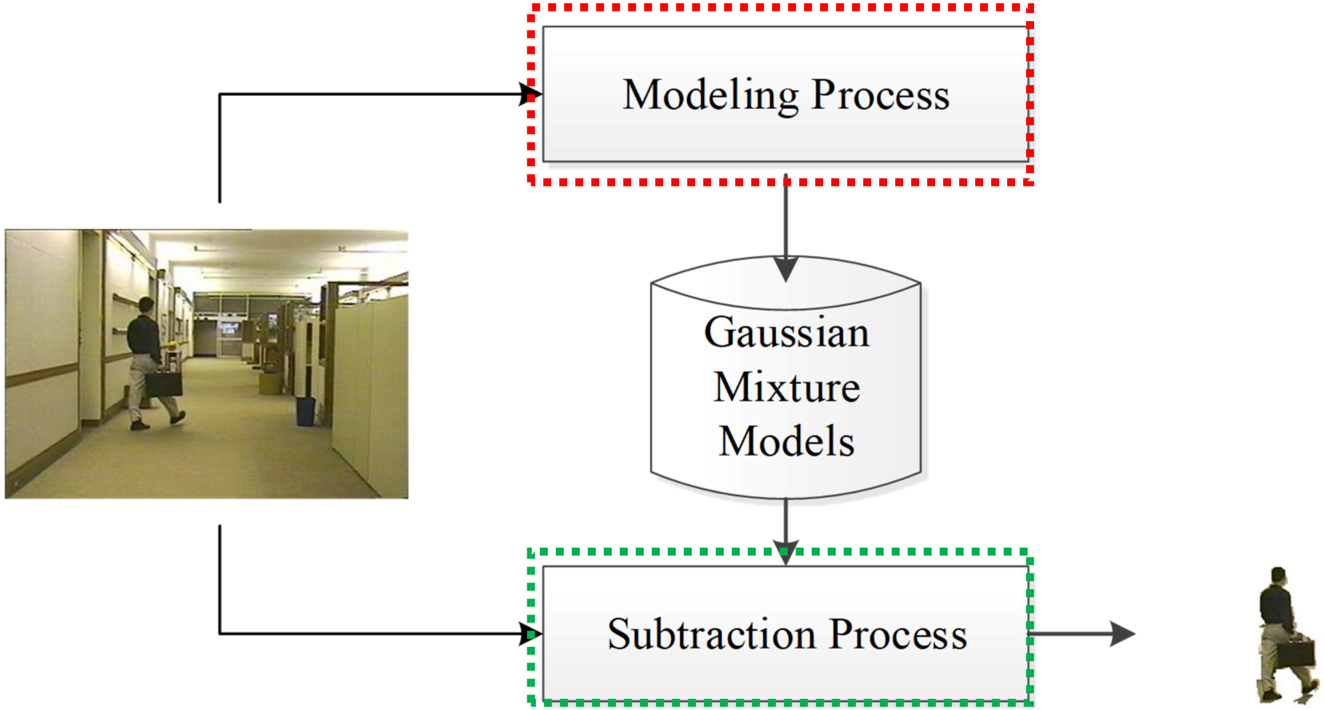
- About Background Subtraction
  - Assumption: camera is stationary
  - Objective: segment the region of interests (foreground) from the background scenes

Original Image

Foreground Mask

Introduction

- Framework

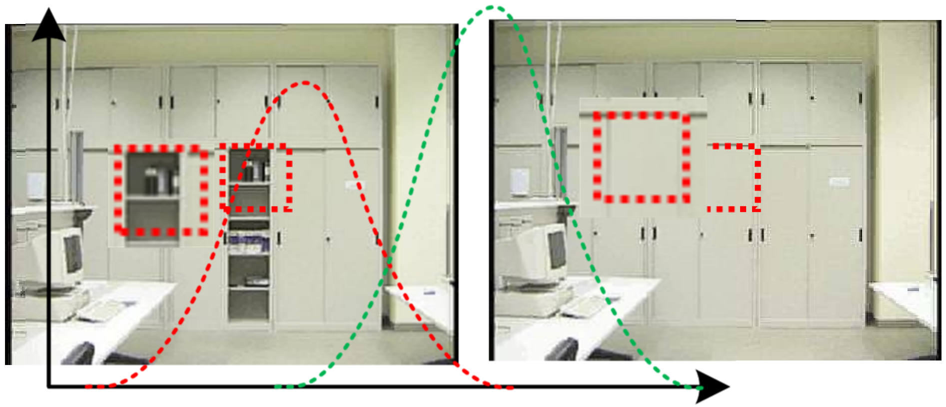


Modeling: describe the background using Gaussian mixture model.

Subtraction: subtract the background from the currently observed image.

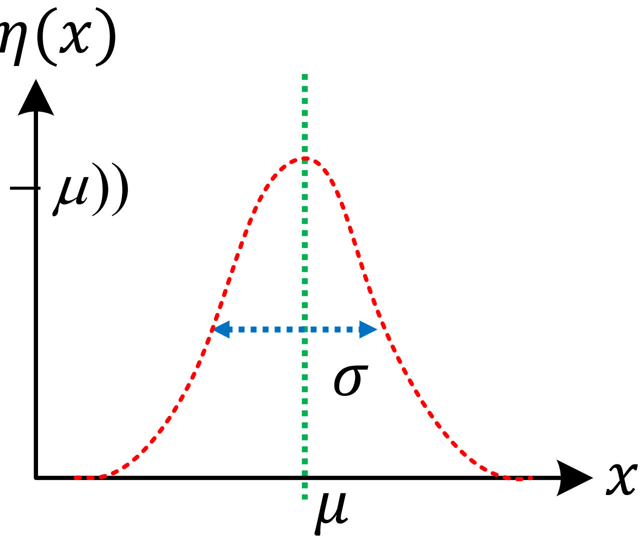
Introduction

- Idea
  - Lighting Variation ➡ **Gaussian Distribution**
  - Appearance Change ➡ **Multiple Gaussians**



Gaussian Mixture Model

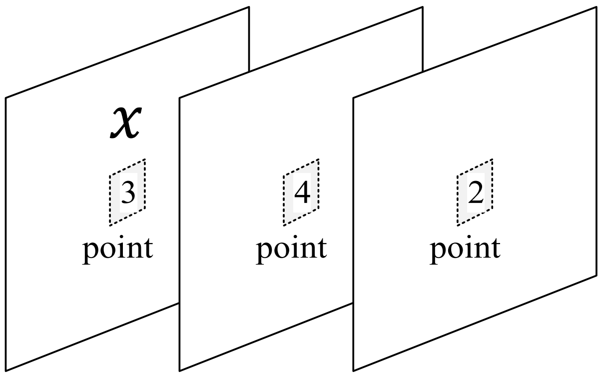
- Gaussian Distribution $\eta(.)$



$$\eta(x;\mu,\sigma)=\frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

$$\eta(x;\mu,\Sigma)=\frac{1}{\sqrt{2\pi|\Sigma|}}\exp\left(-(x-\mu)^T\Sigma^{-1}(x-\mu)\right)$$

  - μ: mean value
  - μ: mean vector
  - $\Sigma=\sigma^2$: variance
  - Σ: covariance matrix



$$\mu=\frac{3.0+4.0+2.0}{3}=3.0 \qquad \sigma=\sqrt{0.67}=0.81$$

$$\eta(x;\mu,\sigma)=\frac{1}{\sqrt{2\pi}\times 0.81}\exp\left(-\left(\frac{x-3.0}{0.81}\right)^2\right)$$

Gaussian Mixture Model

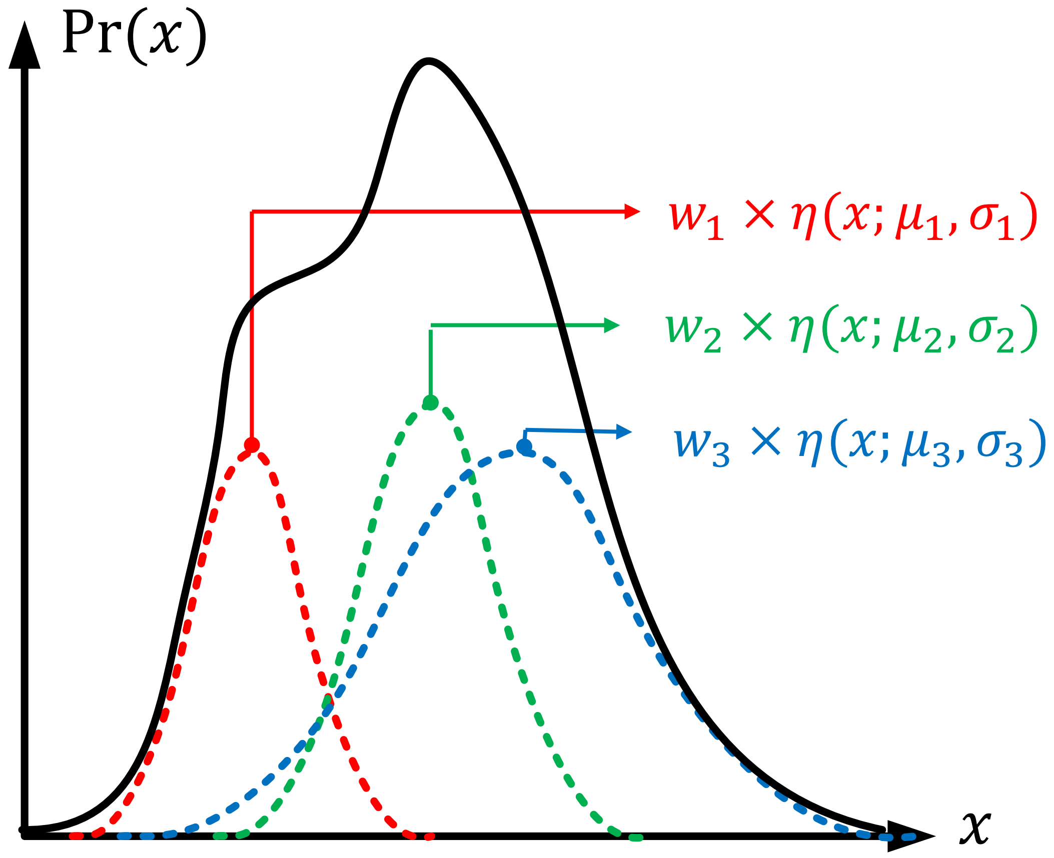
- Definition
  - GMM is a mixture of K Gaussians describing the distribution of a random variable x.

$$\Pr(x) = \sum_{k=1}^{K} w_k \times \eta(x; \mu_k, \sigma_k) \qquad \sum_{k=1}^{K} w_k = 1$$

- w_k: weight of the kth Gaussian
- $\eta(.)$: kth Gaussian distribution with mean value μ_k and standard deviation σ_k

Gaussian Mixture Model

- Example ($K = 3$)

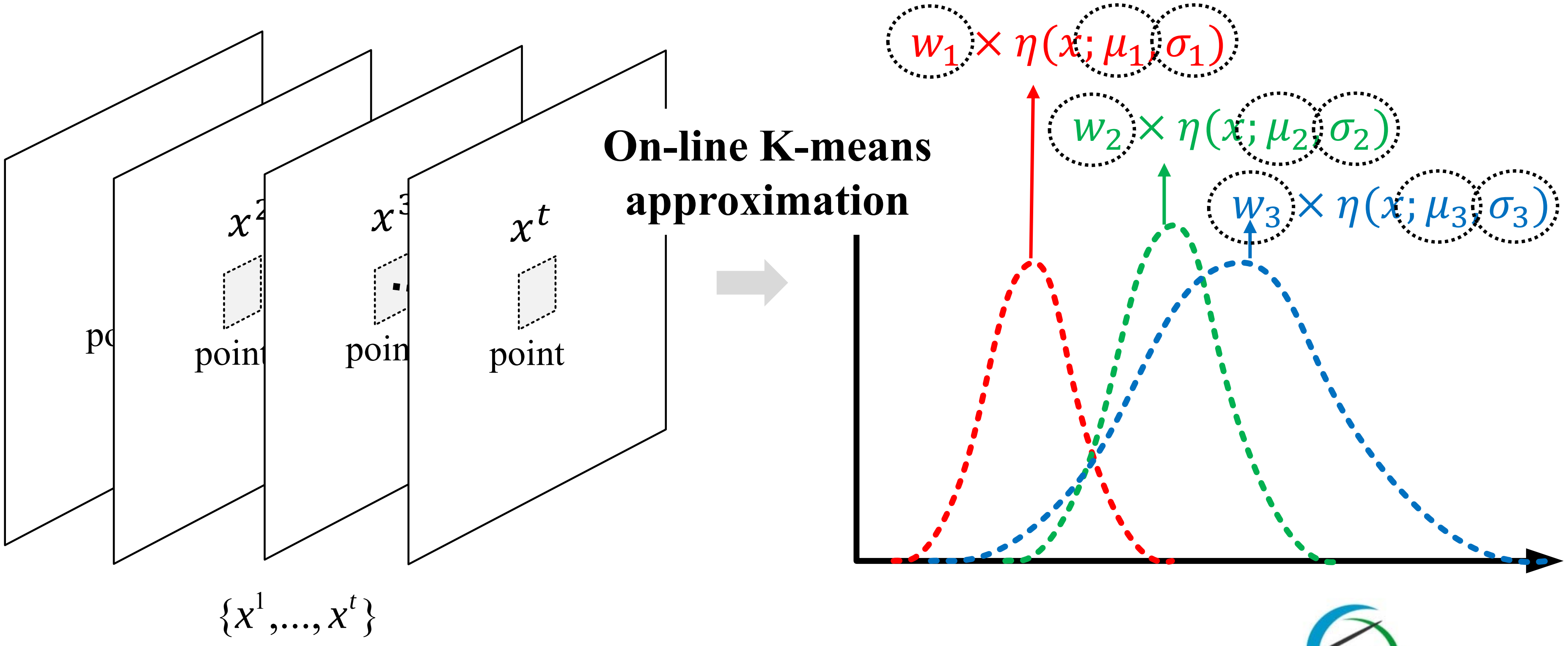


Modeling Process

- Modeling Description
  - All image points are considered as be mutually independent
  - The intensity distribution of every point at any time t is modeled as a GMM.
  - For computation purpose, an on-line K-means approximation is used for modeling.

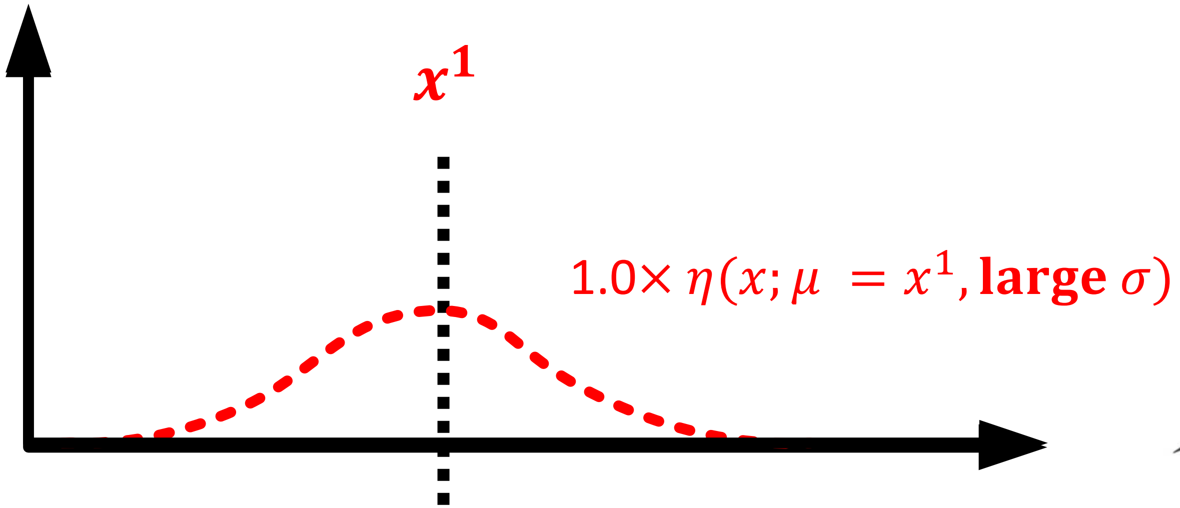
Modeling Process

- Modeling Description



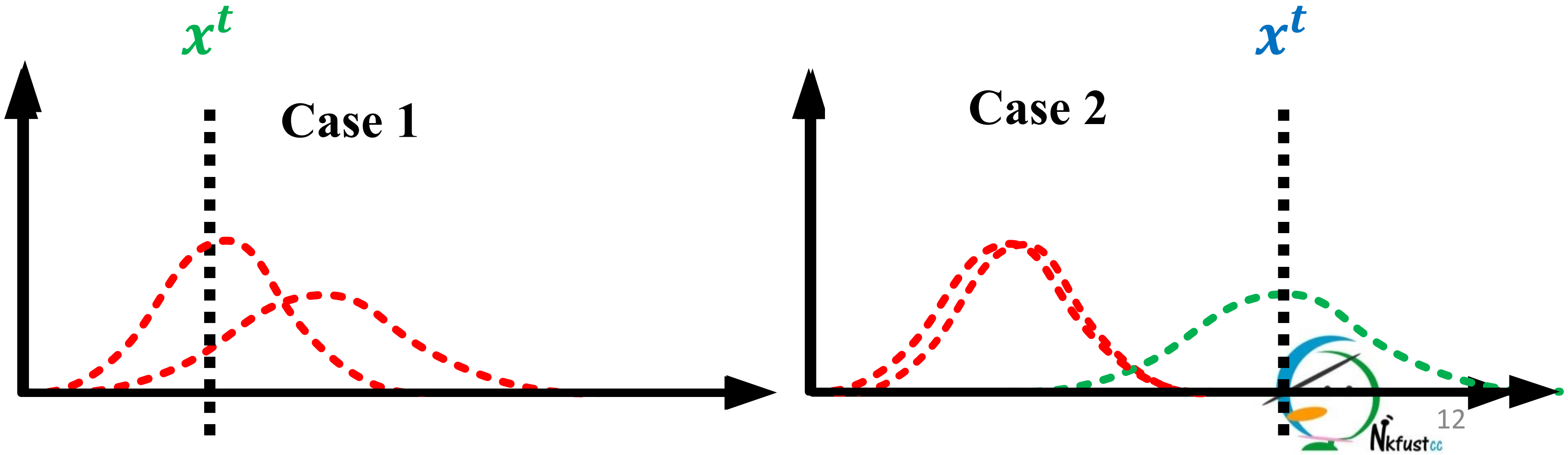
Modeling Process

- On-line K-means approximation
  - Initialization: create a Gaussian η to model first observation x^1
    - $w = 1.0$
    - $\mu = x^1$ and large σ



Modeling Process

- On-line K-means approximation
  - Iterative Step: check if any Gaussians will generate current observation x^t (match)
    - Case 1 (at least one match): update all matched ones
    - Case 2 (no match): create a Gaussian to model x^t



Modeling Process

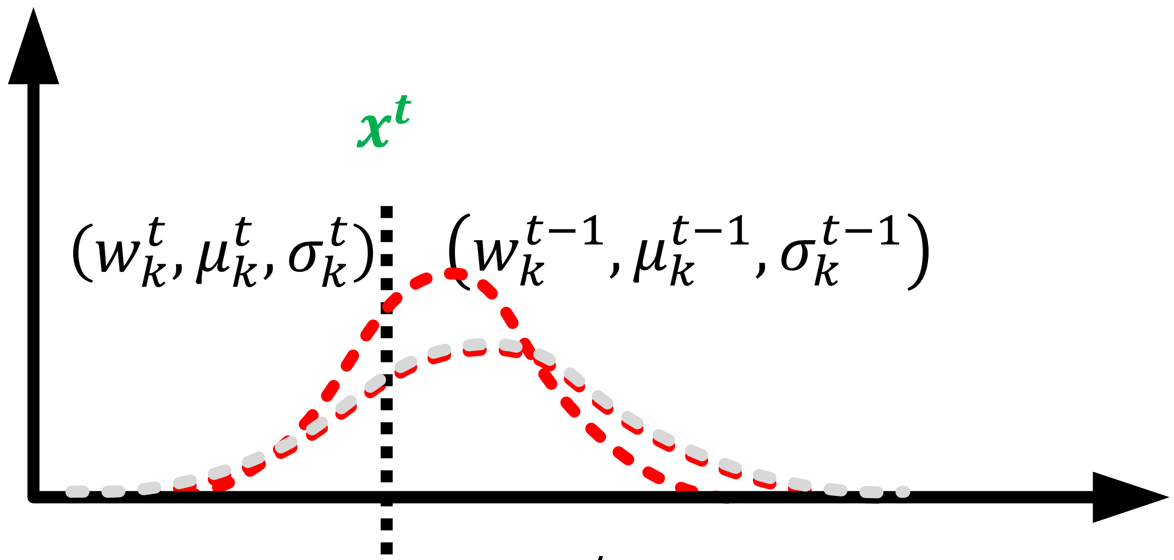
Matching Definition $M_k(x^t)$

- A match of x^t to the kth Gaussian $\eta(x; \mu_k^t, \sigma_k^t)$ is defined as within 2.5 standard deviations.

$$M_k(x^t) = \begin{cases} 1 & if \ \dfrac{(x^t - \mu_k^t)}{\sigma_k^t} \leq 2.5 \\ 0 & \text{otherwise} \end{cases}$$

Modeling Process

- Case 1: Updating Model
  - update parameters of all matched Gaussians by x^t



$$(w_k^{t-1}, \mu_k^{t-1}, \sigma_k^{t-1}) \overset{x^t}{\Rightarrow} (w_k^t, \mu_k^t, \sigma_k^t)$$

Modeling Process

- Case 1: Updating Model

$$x^t = 110$$

$$(w_k^{t-1}, \mu_k^{t-1}, \sigma_k^{t-1}) \; (0.3, 100.0, 16.0) \Rightarrow (w_k^t, \mu_k^t, \sigma_k^t) \; (0.307, \ldots)$$

- Weight: $w_k^t = (1-\alpha)w_k^{t-1} + \alpha$ → **Learning Rate (Constant)**

$\boldsymbol{\alpha}$ **=0.01** ➡ $w_k^t = (1-0.01) \times 0.3 + 0.01$

➡ $w_k^t = 0.307$

Modeling Process

- Case 1: Updating Model

$$x^t = 110$$

$$(0.3, 100.0, 16.0) \Rightarrow (0.307, \mu_k^t, \sigma_k^t)$$

- Mean Value: $\mu_k^t = (1 - \rho_k)\mu_k^{t-1} + \rho_k x^t$

$$\rho_k = \alpha \times \eta(x^t; \mu_k^{t-1}, \sigma_k^{t-1})$$

$$\rho_k = 0.01 \times \eta(x^t = 110; 100.0, 16.0)$$

$$= 0.01 \times \frac{1}{\sqrt{2\pi} \times 16.0} \exp\left(-\left(\frac{110.0 - 100.0}{16.0}\right)^2\right)$$

Fitness

$$\rho_k = 0.0017$$

Modeling Process

- Case 1: Updating Model

$$x^t = 110$$

$$(0.3, 100.0, 16.0) \Rightarrow (0.307, 100.0017, \sigma_k^t)$$

- Mean Value: $\mu_k^t = (1-\rho_k)\mu_k^{t-1} + \rho_k x^t$

$\rho_k = 0.00017$

$$\mu_k^t = (1 - 0.00017) \times 100.0 + 0.00017 \times 110$$

$$= 100.0017$$

Modeling Process

- Case 1: Updating Model

$$x^t = 110$$

$$(0.3, 100.0, 16.0) \Rightarrow (0.307, 100.0017, \sigma_k^t)$$

$$(0.3, 100.0, 16.0) \Rightarrow (0.307, 100.0017, 15.986)$$

- Standard Deviation:

$$(\sigma_k^t)^2 = (1 - \rho_k)(\sigma_k^{t-1})^2 + \rho_k (x^t - \mu_k^t)^2$$

$$(\sigma_k^t)^2 = (1 - 0.00017) \times (16.0)^2 + 0.00017 \times (110 - 100.0017)^2$$

$$\sigma_k^t = 15.986$$

Modeling Process

- Case 2: Creating Model
  - create a new Gaussian if no Gaussians match x^t
    - low weight
    - $\mu = x^t$ and large σ
  - remove the Gaussian with least weight if there are already K Gaussians.
  - add the created Gaussian as one of K Gaussians.

Subtraction Process

- Background Selection
  - sort K Gaussians in a descending order with respect to w/σ
  - select first B Gaussians as background models

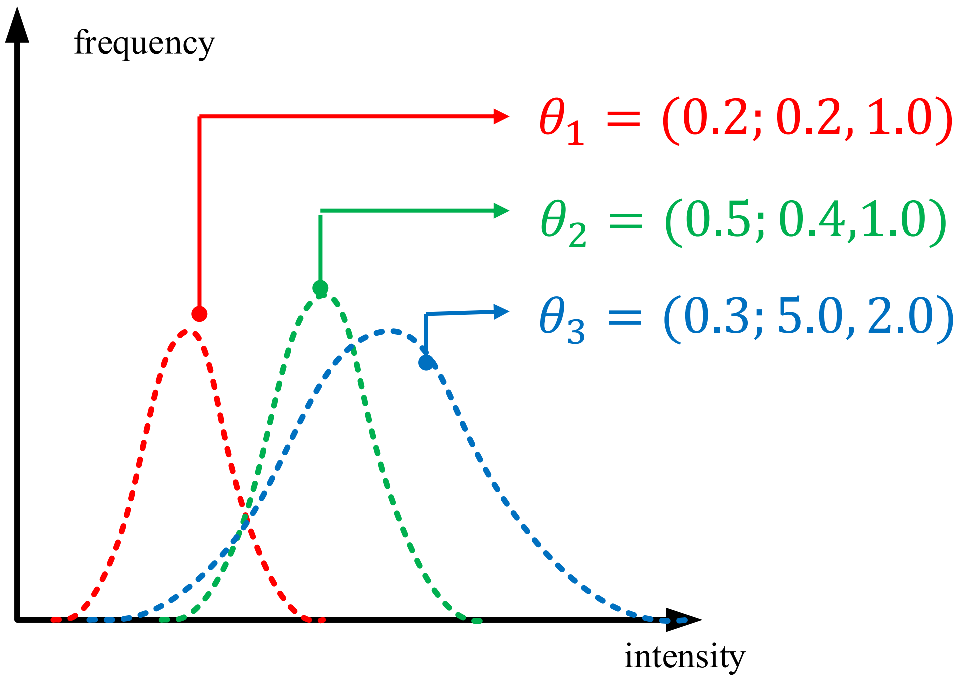
$$B = \arg\min_b \left(\sum_{k=1}^{b} w_k > T_B\right)$$

- T_B: selection threshold

Subtraction Process

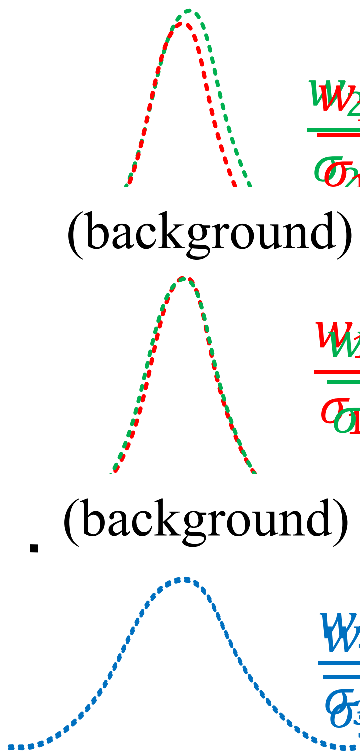
- Background Selection ($T_B = 0.6$)

$$B = \arg\min_b \left(\sum_{k=1}^{b} w_k > T_B\right)$$



sorting

B=2



$\frac{w_1}{\sigma_1} = \frac{0.2}{1.0} = 0.2$

$\frac{w_2}{\sigma_2} = \frac{0.5}{1.0} = 0.5$

(background)

$\frac{w_2}{\sigma_2} = \frac{0.5}{1.0} = 0.5$

$\frac{w_1}{\sigma_1} = \frac{0.2}{1.0} = 0.2$

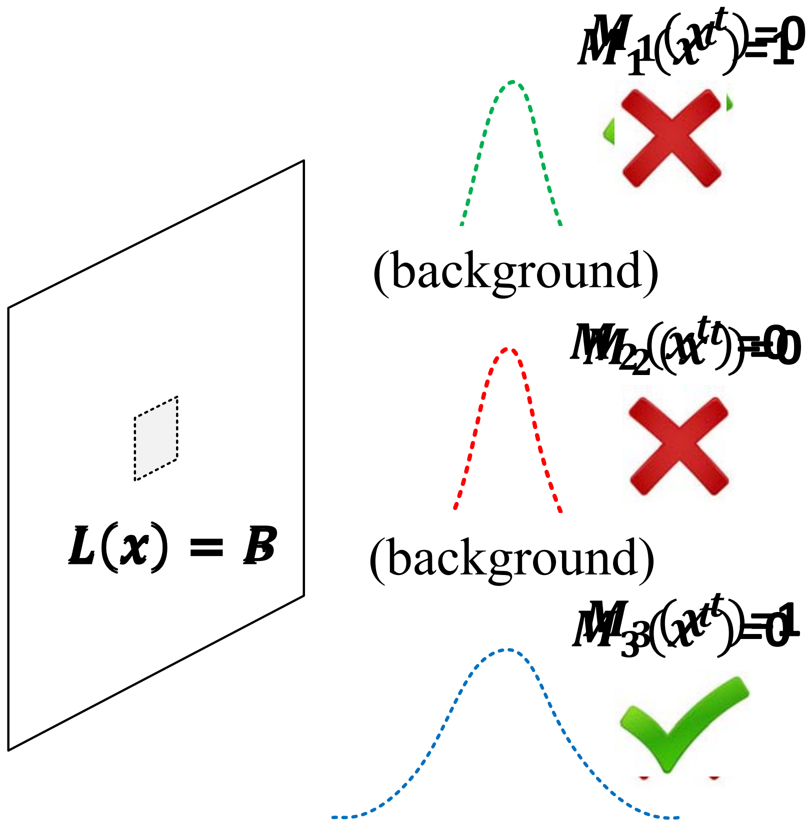
(background)

$\sum w_k = 0.7 > T_B$

$\frac{w_3}{\sigma_3} = \frac{0.3}{2.0} = 0.15$

Subtraction Process

- Point Labelling ($L(x)$)
  - $L(x) = B$: at least one of background Gaussians is matched.
  - $L(x) = F$: none of backgrounds are matched




thank you
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